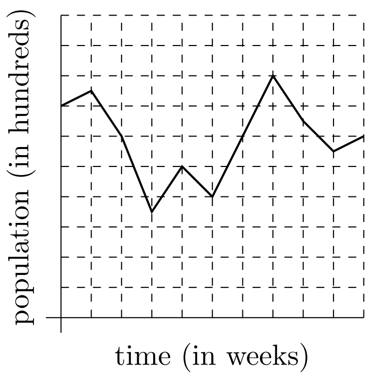
University of Wisconsin – Eau Claire
2026 Math meet
February 21 ♡, 2026

Event 1: Elementary Algebra

Name: ___________________ **School:** ___________________ **Team:** ___________________

Simplify final answers and place them in the space given.

(1) (2 points) Every Saturday at Bryllig you have been traveling into the Turlgey Wood and measuring the population of Jub-Jub Birds. The measurements are encoded in the graph below.



Which of the following conclusions are supported from the graph:

(A) the population continually increases from week 5 through week 7.
(B) the population continually increases from week 4 through week 8.
(C) The maximum population is observed at week 4.
(D) The maximum population is observed at week 8.

Solution: from week 5 to week 6 the population decreases, so B is incorrect. The **minimum** population is at week 4, so C is very incorrect. The population is greatest at week 7, so D is incorrect. This leaves (A).

Answer: __________ A

(2) (3 points) My grade in a certain math class is determined by three tests. My highest test score will count for 50% of my grade and my lowest for 20%; the "middle" test score will count for 30%. I must have a weighted average of 60 in order to pass the class. If I have already earned a score of 70 and 60 on my first two tests, what is the lowest score I can earn on the third test for which I will still pass the class?

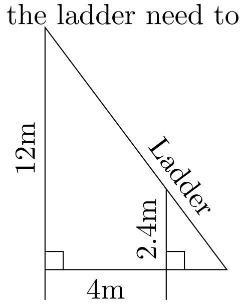
Solution: If the missing grade is greater than or equal to 60, then every grade is at least 60 and so the weighted average will be as well. So the unknown grade, call it ℓ is the lowest. Thus, we must solve

$$\text{grade} = 0.5 \cdot 70 + 0.3 \cdot 60 + 0.2 \cdot \ell \quad = \quad 60$$

Doing so and writing a few steps, $35 + 18 + 0.2\ell = 60$, $\frac{1}{5}\ell = 7$, $\ell = 7 \cdot 5 = 35$,

Answer: __________ 35

(3) (4 points) The brave Princess Algebraica wishes to save a knight who is trapped in a tower. The tower forms a right angle with the ground. She needs a ladder tall enough to go over a 2.4-meter tall thorny hedge placed 4 meters from the base of the tower and reach the tower window, which is 12 meters above the ground. How long does the ladder need to be?



Solution: Let x be the length from the princess to the base of the hedge. You now see two similar triangles, so that $\frac{x}{2.4} = \frac{x+4}{12}$. Solve this for x. $12x = 2.4x + 2.4 \cdot 4$. $(12 - 2.4)x = 9.6$. $9.6x = 9.6$, Thus, $x = 1$.

Now use the Pythagorean formula to get $\ell^2 = 12^2 + 5^2$. The astute amongst you will note that $(5, 12, 13)$ is a Pythagorean triple, so $\ell = 13$. Otherwise, you get $\ell^2 = 144 + 25 = 169$, so $\ell = \sqrt{169} = 13$. Don't forget to include units! $\boxed{\text{13 meters}}$

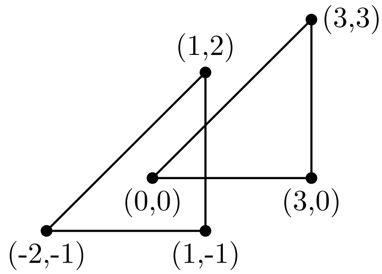
Answer: ___________ 13 meters

Event 2: Geometry

Name: ____________________ **School:** ____________________ **Team:** ____________________

Simplify final answers and place them in the space given.

(1) (2 points) Below you see two overlapping right triangles. What is the total area they cover?



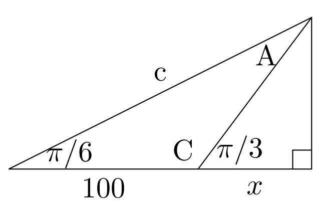
Solution: Each triangle has area $\frac{\text{base}\cdot\text{height}}{2} = \frac{9}{2}$. The small triangle in their intersection has base length $1 - 0 = 1$ and height equal to its base.
The inclusion-exclusion principle now says that area $= \frac{9}{2} + \frac{9}{2} - \frac{1}{2} = \frac{17}{2}$.

Note: The inclusion-exclusion principle, sometimes called the double counting principle is the observation that when yo sum up two areas, you have double counted their overlap. So if you "uncount" (subtract) the overlap, you are left with the area they cover

Answer: __________ $\frac{17}{2}$ or 8.5

(2) (3 points) Princess Algebraica spies a dragon perched atop a tower. Using her trusty protractor, she determines that the dragon's angle of elevation (the angle between the dragon, her eyes, and a horizontal line from her eyes to the tower) is $\frac{\pi}{6}$, or 30^o. Cautiously, she advances 100 meters closer to the tower. Now the dragon's angle of elevation is $\frac{\pi}{3}$, or 60^o. How much farther does Princess Algebraica need to go to reach the tower?

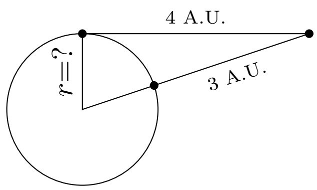
Solution: The scenario can be represented as follows, where x is the unknown length she still must walk.



By complementary angles, $C = \frac{2\pi}{3}$. Because the angles of a triangle sum to π, $A = \frac{\pi}{6}$. By the Law of Sines, $\frac{\sin\frac{\pi}{6}}{100} = \frac{\sin\frac{2\pi}{3}}{c}$, so $\frac{1/2}{100} = \frac{\sqrt{3}/2}{c}$. This means that $c = 100\sqrt{3}$. Then, using the larger right triangle, $\cos\frac{\pi}{6} = \frac{100+x}{c}$, so $\frac{\sqrt{3}}{2} = \frac{100+x}{100\sqrt{3}}$, which gives $\frac{3\cdot 100}{2} = 100 + x$. Therefore, $x = \frac{1}{2}\cdot 100 = 50$ meters.

Answer: __________ 50 meters

(3) (4 points) While flying in a spaceship you spot a distant spherical megastructure. By measuring the time it takes for light you emit to reflect back at you, you estimate the distance from you to the nearest point on the planet to be 3 astronomical units (AU) and the distance from you to its north pole to be 4 $A.U.$ (See the picture below). Determine the radius of the megastructure, in $A.U.$[1] (You may express your answer either as a simplified fraction or as a decimal rounded to the nearest tenth of an $A.U.$)



Solution: You see here a right triangle with hypotenuse $3+r$, one leg 4 and one equal to r. (We are suppressing the unit.) By the pythagorean theorem, $4^2+r^2=(3+r)^2$. So $16+r^2=9+6r+r^2$. Solving, $r=\frac{7}{6}\approx 1.2$ A.U.

Answer: ___________ $\frac{7}{6}$ A.U. or 1.2 A.U.

[1]One astronomical unit is about 150 million kilometers. It is the average distance from the earth to the sun.

Event 3: Intermediate Algebra

Name: ____________________ **School:** ____________________ **Team:** ____________________

Simplify final answers and place them in the space given.

(1) (2 points) You randomly select a date from this year's calendar. What is the probability that the date you select is the 30'th of a month? Enter your answer as a fraction in reduced form. Hint: This year is not a leap-year.

Solution: There are 365 dates this year. All but February have a 30'th, so $\frac{11}{365}$ of them are the 30'th of that month.

Answer: __________ $\dfrac{11}{365}$

(2) (3 points) Princess Algebraica comes across a village of friendly goblins in her adventures. The total population of this village is 500. Amongst those, fifty have both horns and a tail, the number of goblins with a tail is double the number with horns, the number with tail and no horns is triple the number with horns and no tail. How many goblins have neither horns nor a tail?

Solution: Let $x =$ (number with a tail) and $y =$ (number with horns), so that $x - 50 =$ (number with a tail and no horns) (take all of them with a tail and remove those with both horns and a tail) and $y - 50 =$ (number with horns and no tail). Thus:

$$x = 2y \text{ and } x - 50 = 3(y - 50)$$

Plug $x = 2y$ into the second equation: $2y - 50 = 3y - 150$, so $100 = y$. Thus, $x = 2y = 200$. Now we break the population up into four categories:

- Horns and a tail: 50
- Tail and no horns: $x - 50 = 150$
- Horns and no tail: $y - 50 = 100 - 50 = 50$
- No horns and no tail: The rest of them: $500 - 50 - 150 - 50 = \boxed{50}$

Answer: __________ 50 or 50 goblins

(3) (4 points) $p(x) = ax^2 + bx + c$ is a degree 2 polynomial with unknown coefficients. Suppose that $p(x)$ satisfies the table of values below.

x	0	1	3	5
$p(x)$	2	0	14	52

Determine $p(2)$.

Solution: Use $2 = p(0) = a \cdot 0^2 + b \cdot 0 + c$ to conclude $c = 2$.

Now use $p(1) = 0$ and $p(3) = 14$ to conclude $\begin{bmatrix} 0 & = & a + b + 2 \\ 14 & = & 9a + 3b + 2 \end{bmatrix}$, so $\begin{bmatrix} -2 - a & = & b \\ 12 & = & 9a + 3b \end{bmatrix}$. We can now plug $b = -2 - a$ in to $12 = 9a + 3b$ to get $12 = 9a - 6 - 3a$, so $18 = 6a$ and $a = 3$. Then $b = -2 - a = -5$. We now have $p(x) = 3x^2 - 5x + 2$.

Plug in $x = 2$: $p(2) = 3 \cdot 4 - 5 \cdot 2 + 2 = 12 - 10 + 2 = 4$.

Answer: ___________ 4

Event 4: Advanced Mathematics

Name: ___________________ **School:** ___________________ **Team:** ___________________

Simplify final answers and place them in the space given.

(1) (2 points) Determine the one's digit in 2026^{2026}

Solution: To build intuition think about low powers of 2026:

$$2026^2 = (2020+6)\cdot(2020+6) = 2020^2 + 2\cdot 2020\cdot 6 + 36$$

Notice that only the "36" contributes to the ones place. In fact, in general, the one's digit in a product is determined by the one's digits in the factors, so the one's digit of 2026^{2026} is the same as the one's digit of 6^{2026}.

n	2	3	4
6^n	36	216	1296

The one's digit never changes from 6. So the one's digit of 2026^{2026} is 6.

Answer: __________ 6

(2) (3 points) If x is an arbitrary integer, what is the largest possible remainder when $9x$ is divided by 30?

Solution: Remainders must always be less than the divisor, so we only need to consider remainders less than or equal to 29. When $x = 3$, $9x = 27$. When this is divided by 30, we get 0 with a remainder of 27, so 27 is possible. Thus the answer is either 27, 28, or 29. To show that a remainder of 28 is not possible, notice that for $9x/30$ to have a remainder of 28, there must be some quotient q such that $9x = 30q + 28$. Then $9x - 30q = 28$. But $9x - 30q = 3(3x - 10q)$ is divisible by 3, and 28 is not divisible by 3. Therefore, it is impossible to get a remainder of 28. A similar argument shows that it is impossible to get a remainder of 29. Thus, the maximal remainder is 27.

Answer: __________ 27

(3) (4 points) A dragon has kidnapped the town mayor and imprisoned her in a cage. To taunt Princess Algebraica, the dragon has placed the key to the cage atop a 12-foot tall wall, at point $(8, 12)$. Due to the moat on the other side of the wall, Princess Algebraica can only get within 8 feet of the wall, so she is standing at point $(0, 0)$. Princess Algebraica quickly loads a sticky piece of gum into her slingshot. She then shoots the gum in a parabola, $y = ax^2 + bx + c$, so that it touches (and sticks to) the key and then continues to the mayor at position $(10, 0)$, allowing the mayor to unlock her cage. What is a? Give your answer as a fraction in lowest terms.

Solution: $a = \frac{-3}{4}$. The parabola has roots at $x = 0$ and $x = 10$, so it must have the form $a(x-10)x$. The point $(8, 12)$ is on the parabola, so we have $a(8-10)\cdot 8 = 12$. This simplifies to $a(-2)\cdot 8 = 12 \Rightarrow a\cdot -16 = 12$, so $a = -12/16 = -3/4$.

Answer: __________ $-3/4$

Team Event

School: ____________________ **Team:** ____________________

Simplify final answers and place them in the space given.

(1) (10 points) Compute $\dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}+\dfrac{1}{3\cdot 4}+\cdots+\dfrac{1}{2025\cdot 2026}$. Return your answer as a fraction in reduced form.

Solution: We will use the following idea: $\frac{1}{i\cdot(i+1)}=\frac{i+1-i}{i\cdot(i+1)}=\frac{1}{i}-\frac{1}{i+1}$ so we can replace this sum with

$$\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{2024}-\frac{1}{2025}\right)+\left(\frac{1}{2025}-\frac{1}{2026}\right).$$

All of the terms except for the $\dfrac{1}{1}$ from the first summand and the $\dfrac{1}{2016}$ from the last summand cancel, leaving us with

$$1-\frac{1}{2026}=\frac{2025}{2026}$$

Answer: __________ $\frac{2025}{2026}$

(2) (10 points) Find the largest five digit number such that if you subtract it from the number formed by switching its first and last digits, you get a positive number divisible by 22 but not by 44.

Solution: Let the required number be

$$x := a_4 10^4 + a_3 10^3 + a_2 10^2 + a_1 + a_0$$

The number formed by switching its first and last digits is

$$y := a_0 10^4 + a_3 10^3 + a_2 10^2 + a_4$$

Subtracting x from y we get

$$\begin{aligned} y-x &= (a_0-a_4)10^4-(a_0-a_4)\\ &= (a_0-a_4)(10^4-1)\\ &= (a_0-a_4)(9999)\\ &= (a_0-a_4)3^2\cdot 11\cdot 101 \end{aligned}$$

Digits a_1, a_2, a_3 do appear in this constraint, so we let them all be 9. We want a_4 and a_0 to be as large as possible with a_0-a_4 positive and divisible by 2 but not by 4. $a_4=9$ wouldn't work, since the difference would then be negative or zero. The next largest choice is $a_4=7$ and then the largest choice for a_0 would be $a_0=9$. Thus, the required number has to be 79999.

Answer: __________ 79999

(3) Princess Algebraica wants to visit a friendly and very honest troll, who lives in one of six caves labeled from east to west as (a), (b), (c), (d), (e), or (f). However, she discovers that 5 unfriendly ogres have moved into the other five caves. Ogres are known for decorating their caves with signs, which are always untrue. Princess Algebraica encounters 6 caves with the following signs:

(a) The troll is in this cave.
(b) The troll is in cave **e**.
(c) Caves **c** and **d** contain the same species of creature.
(d) The troll is in this cave.
(e) All of caves **a** through **d** contain ogres.
(f) Exactly one of caves **a** or **b** contains a troll.

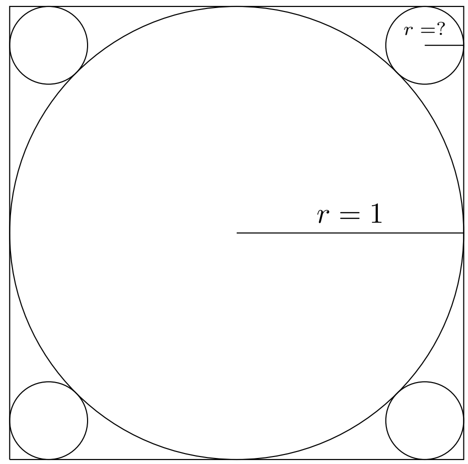
In which cave does the troll live?

Solution: The troll lives in cave **d**. To determine this, start by considering statement **e**. Either the statement is true (in which case the troll lives in cave **d**), or it is false (in which case the troll lives in one of caves **a** through **d**). Either way, cave **f** cannot contain the troll. Therefore, the statement "exactly one of caves **a** or **b** contains a troll" is false. This means that caves **a** and **b** either both contain trolls, or both contain ogres. There is only 1 troll, so caves **a** and **b** must both contain ogres. Therefore, the statement on cave **b**, "The troll is in cave **e**" is false. We now know that caves **a, b, e**, and **f** all contain ogres; the troll is in either cave **c** or **d**. This means that the statement on cave **c**, "Caves **c** and **d** contain the same species of creature," is false. Therefore, cave **c** contains an ogre, so the troll lives in cave **d**.

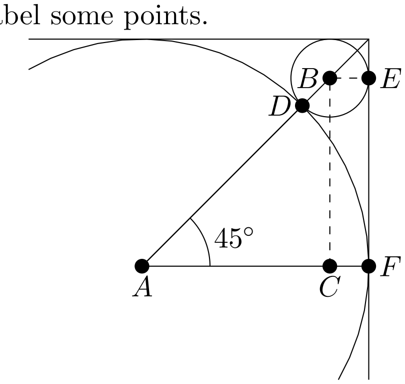
Answer: __________ d, or cave d

(4) Greta's garden sits in a 2×2 square. It consists of one large circle inscribed in this square as well as four smaller circles, each of which is simultaneously tangent to the big circle and to the square. See the figure below.

What is the radius of these smaller circles? Remember to simplify your answer.



Solution: Let x denote the unknown radius of the small circles. Zoom in on the diagram, draw a dashed line from the center of the small circle perpendicular to an edge of the square and label some points.



Observe that Δ_{ABC} is a right triangle with angle $\measuredangle CAD = 45°$. We will find relationships amongst its side lengths. Its hypotenuse is a radius of the big circle 1 with a radius of the small circle, So the hypotenuse is length $1 + x$.

The side length AC with the side length CF totals up to the radius of the big circle. Using that $\square(CFEB)$ is a rectangle, $|CF| = x$. Thus $|AC| = 1 - x$.

We now use that $\cos(45°)$ is $\frac{|AC|}{|CB|}$ to get $\dfrac{1}{\sqrt{2}} = \dfrac{1-x}{1+x}$. We solve for x,

$$\begin{aligned} \frac{1}{\sqrt{2}} &= \frac{1-x}{1+x} \\ 1+x &= \sqrt{2}(1-x) \\ (\sqrt{2}+1)x &= \sqrt{2}-1 \\ x &= \tfrac{\sqrt{2}-1}{\sqrt{2}+1} \end{aligned}$$

To rationalize the denominator, we multiply and divide by $\sqrt{2} - 1$,

$$x = (\sqrt{2}-1)^2 = 3 - 2\sqrt{2}$$

Answer: ___________ $3 - 2\sqrt{2}$

(5) Determine the exact value of

$$\sqrt{3 + \sqrt{3 + \sqrt{3 + \ldots}}}.$$

Simplify your answer.

Solution: Let $x = \sqrt{3 + \sqrt{3 + \sqrt{3 + \ldots}}}$. Then $x = \sqrt{3 + x}$. Then $x^2 - x - 3 = 0$, so by the quadratic formula, $x = \frac{1 \pm \sqrt{13}}{2}$. x is positive, so, $x = \frac{1+\sqrt{13}}{2}$. (This is a really powerful trick in mathematics, if you want to study a "self-similar" thing, then start by giving the thing a name and seeing how it "fits into itself".)

Answer: ___________ $\frac{1+\sqrt{13}}{2}$

(6) (10 points) How many digits long is the base 10 decimal representation of 2026^{2026}?

Solution: The number of digits in a number base 10 is given by $\lfloor \log_{10}(\text{that number}) \rfloor + 1$. Here $\lfloor - \rfloor$ is the "round down" function. So we need to compute $\log_{10}(2026^{2026})$. You will notice that your calculator fails to compute this, since 2026^{2026} is too big. By logarithm laws, $\log_{10}(2026^{2026}) = 2026 \cdot \log_{10}(2026) \approx 6699.3$. Rounding down and adding one gives 6700.

Answer: ___________ 6700 or 6700 digits