

University of Wisconsin – Eau Claire  
2020 Math meet  
February 15, 2020

**Event 1: Elementary Algebra**

**Name:** \_\_\_\_\_ **School:** \_\_\_\_\_ **Team:** \_\_\_\_\_

Simplify final answers and place them in the space given.

- (1) (2 points) The Irvine Park Zoo transferred 12.5% of its bison to Yellowstone National Park. Then the Irvine Park Zoo then received a donation of 4 bison from another zoo. Amazingly, the Irvine Park Zoo ended up with the same number of bison it had when it started. How many bison are in the Irvine Park Zoo?

Answer: 32\_\_\_\_\_

- (2) (3 points) A classroom has 5 identical math books and 20 identical science books (and no other books). If the classroom had 20 math books and 5 science books instead, then the total number of pages would be twice as big as it is now. Each science book has 100 pages. How many pages are in one math book?

Answer 350\_\_\_\_\_

- (3) (4 points) Eight consecutive whole numbers add up to 2020. What is the least of these eight numbers?

Answer: 249

**Event 2: Geometry**

**Name:** \_\_\_\_\_ **School:** \_\_\_\_\_ **Team:** \_\_\_\_\_

Simplify final answers and place them in the space given.

- (1) (2 points) If the lengths of each side of a rectangle are tripled, then by what factor will the area be increased?

Answer: 9

- (2) (3 points) A rectangle has perimeter 16 centimeters and diagonal  $\sqrt{34}$  centimeters . What is its area?

Answer: 15 square centimeters.

- (3) (4 points) A standard running track is 400 meters consisting of two 100 meter straight-aways and two half circles each with length 100 meters, put together as in the figure below. What is the area enclosed by a standard track?



Answer:  $\frac{30,000}{\pi}$  square meters.

**Event 3: Intermediate algebra**

**Name:** \_\_\_\_\_ **School:** \_\_\_\_\_ **Team:** \_\_\_\_\_  
Simplify final answers and place them in the space given.

- (1) (2 points) How many solutions to  $\sin(2020x) = \frac{1}{2019}$  are in the interval  $[0, 2\pi]$ ?

Answer: 4040

- (2) (3 points) The country of Mathonia issues currency in two denominations: \$3 and \$7. You are visiting Mathonia and wish to buy a souvenir, whose price is a whole number of dollars. But you find that you cannot pay the souvenir's exact price with integer numbers of Mathonian bills! What is the maximum possible price of the souvenir?

Answer: \$11.

- (3) (4 points) What is the digit in the one's place of  $2^{2020}$ ?

Answer: 6

**Event 4: Advanced Mathematics**

**Name:** \_\_\_\_\_ **School:** \_\_\_\_\_ **Team:** \_\_\_\_\_

Simplify final answers and place them in the space given.

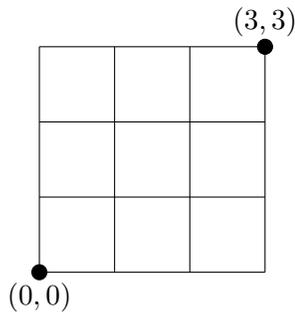
- (1) (2 points) What is the exact value of  $\prod_{k=2}^{2020} \log_k(k+1)$ ?

Answer:  $\log_2(2021)$  or  $\frac{\log(2021)}{\log(2)}$  (Still OK if they specify a base)

- (2) (3 points) Suppose that  $x^2 + xy + y^2 = 84$  and  $x - \sqrt{xy} + y = 6$ . What is the value of  $xy$ ?

Answer: 16.

- (3) (4 points) You start at the point  $(0,0)$  on the grid below. You flip a fair coin six times. On a “heads” you travel up a length of 1. On a “tails” you go right a length of 1. What is the probability that you will end at the point  $(3,3)$ ? Give your answer as a fraction in reduced terms.



Answer:  $\frac{5}{16}$ .

**Team Event****School:** \_\_\_\_\_ **Team:** \_\_\_\_\_

Simplify final answers and place them in the space given.

- (1) (10 points) In the country of Mathonia, 80% of people like math. The probability that a person is cool, given that they like math, is five times the probability of a person being cool, given that they don't like math (just like in our country!) If 63% of all people are cool, what percentage of people who like math are cool?

Answer: 75%. An answer of .75 is incorrect.

- (2) (10 points) Chris's friend used PayVen to send Chris some money, but his friend accidentally entered the number of dollars as the number of cents, and vice versa. After Chris spent 68 cents, he discovered that he still had precisely twice as much money as his friend had originally planned to send him. What is the smallest (positive) amount, in dollars and cents, that Chris's friend could have originally planned to send?

Answer: \$10.21

\$ \_\_\_\_\_ . \_\_\_\_\_

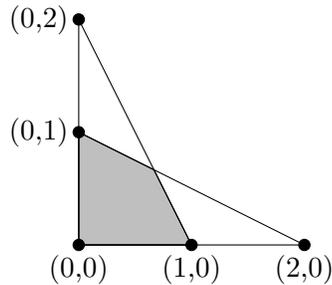
- (3) (10 points) Suppose you have a sequence of three numbers,  $a_1, a_2, a_3$ , whose first term is positive, so that any given term is the perimeter of a square whose area is the preceding term, and which forms an arithmetic progression. (The words "arithmetic progression" mean that  $a_2 - a_1 = a_3 - a_2$ ). Determine all possible values of the third term. Give exact values. Decimal approximations will not receive credit.

Answer: 16 and  $8\sqrt{5} - 8$

- (4) (10 points) Chris is drinking coffee out of a 10 fluid ounce mug. This cup of coffee contains 200 mg of caffeine. Each sip he takes drains 1 fluid ounce. After each sip a goblin refills the mug by adding one fluid ounce of hot water and mixing thoroughly. After Chris has taken ten sips from his coffee, how much of the original 200 mg of caffeine has he ingested? Give either an exact answer in milligrams or a decimal approximation in milligrams rounded to the nearest tenth of a milligram.

Answer:  $\frac{200 - 200 \cdot (9/10)^{10}}{\text{Any of the above is acceptable}}$  mg or  $200 \cdot (1 - (9/10)^{10})$  mg or 130.3 mg

- (5) (10 points) What is the area of the shaded region below?



Answer:  $\frac{2}{3}$  or  $\bar{6}$

- (6) (10 points) Consider any natural number  $v$ . Take this number and square it 100 times to get

$$\underbrace{(\dots((v^2)^2)\dots)^2}_{100 \text{ times}}$$

What could the one's digit in the resulting number possibly be? Give all possibilities.

Answer: 0, 1, 5, 6