

University of Wisconsin – Eau Claire
2019 Math meet
February 16, 2019

Event 1: Elementary Algebra

Name: _____ **School:** _____ **Team:** _____

Simplify final answers and place them in the space given.

- (1) (2 points) A devious cat has learned how to make its owner's alarm clock go off early and it using this to get early breakfast. Each morning the cat has the alarm go off 5 minutes earlier than the morning before. If the alarm was initially set for 6 A.M. then how many nights will pass before the alarm goes off at 12:05 AM?

Answer: 71

- (2) (3 points) Chris has spilled 8 cups of milk and has begun crying over his mess as he cleans it up. His tears contribute an additional 0.5 cups per minute of liquid to the mess. He is able to clean at a rate of 4 cups per minute. How many minutes will it be until the milk and tears are all cleaned up? Report your answer as a fraction in reduced terms.

Answer: $\frac{16}{7}$ OR $2 + \frac{2}{7}$

- (3) (4 points) The minute hand on a standard clock travels around the clock 12 times as fast as the hour hand. How many times between midnight and noon (excluding midnight and noon) do the minute and hour hand point in either exactly the same or exactly the opposite direction? Notice that the problem does not ask what the times are, only how many times there are.

Answer: 23 times

Event 2: Geometry

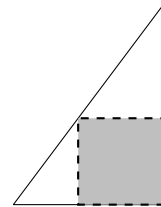
Name: _____ **School:** _____ **Team:** _____

Simplify final answers and place them in the space given.

- (1) (2 points) The U-Transport company sells cube-shaped boxes in 2 sizes: small and medium. Three small boxes placed side-by-side have exactly the same width as two medium boxes placed side-by-side. If the volume of one small box equals 800 packing peanuts, what is the volume of one medium box (as measured in packing peanuts)?

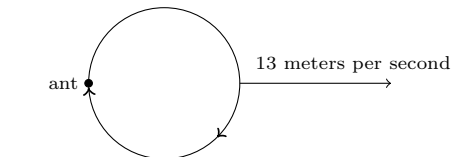
Answer: 2700

- (2) (3 points) Start with a right triangle with length 3 and height 4. Draw inside of this triangle a square having one corner at the right angle of the triangle and another on its hypotenuse. What is the area of this square?



Answer: $\frac{144}{49}$ OR $\left(\frac{12}{7}\right)^2$ OR 2.94

- (3) (4 points) A car is traveling at a speed of 13 meters per second down a road. The wheels of this car have diameter 50 centimeters, or 0.5 meters. An ant is clinging to the edge of the tire. When the ant is at the back of the tire how fast is the ant traveling in meters per second. Simplify your answer.



Answer: $13\sqrt{2}$

Event 3: Intermediate algebra**Name:** _____ **School:** _____ **Team:** _____

Simplify final answers and place them in the space given.

- (1) (2 points) A dry sponge weighing 8 ounces is placed in water. It is removed once it is 90% water by weight. How much water in ounces does the sponge hold when it is removed?

Answer: 72 ounces

- (2) (3 points) Suppose that b and P are numbers for which $\log_b(P + P) = 3 \cdot \log_b(P) = \log_b(P) + 1$. Determine both b and P .

 $b=2, P = \sqrt{2}$ OR $2^{1/2}$.

- (3) (4 points) Solve the equation $\sqrt{x} + \sqrt{x+1} - \sqrt{4x + \frac{3}{2}} = 0$. Simplify your answer.

Answer: $\frac{1}{8}$ or .125

Event 4: Advanced Mathematics**Name:** _____ **School:** _____ **Team:** _____

Simplify final answers and place them in the space given.

- (1) (2 points) The name of the current month “February” can be rearranged to “Fur by ear.” How many ways are there to rearrange the letters in “February” if you are not allowed to change the order of the vowels? That is “e” must come before “u” must come before “a” must come before “y.”

Answer: 840.

- (2) (3 points) a and b are two positive integers whose squares differ by 2019. What is the farthest apart that a and b can be? (Hint: 673 is prime.)

Answer: 3

- (3) (4 points) Let a , b , and c be numbers. Suppose that $a+b+c = 10$ and $a^2+b^2+c^2 = 10$. What is $ab + bc + ca$?

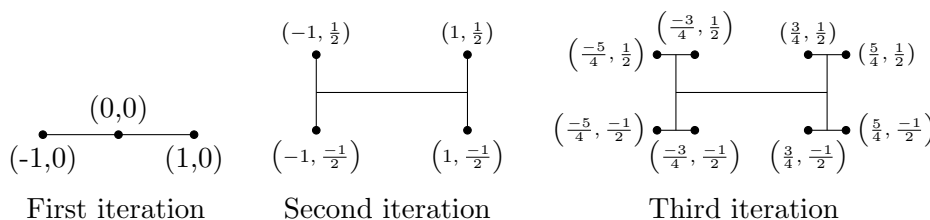
Answer: 45

Team Event

School: _____ **Team:** _____

Simplify final answers and place them in the space given.

- (1) (10 points) Begin with a single point at the origin $(0,0)$ shoot out line segments of length 1 to the left and right to get two new points, $(1,0)$ and $(-1,0)$. From each of these two points send out lines of length $1/2$ going up and down. From these four points send out lines of length $1/4$ to the left and right. Continue this process 100 times, alternating between the horizontal and vertical directions and halving the lengths of the new lines. The first three iterations are depicted below. What is the distance from the point furthest from the origin to the origin? Report your answer up to the fifth digit after the decimal.



Answer: 1.49071

- (2) (10 points) Let $F(x) = \cos(\pi x)$ and F^n be the function given by composing F with itself n times. Thus, $F^3(x) = \cos(\pi \cos(\pi \cos(\pi x)))$. How many solutions to the equation $F^{10}(x) = \frac{19}{20}$ are there in the interval $[-1, 1]$?

Answer: 2^{10} or 1024

- (3) (10 points) A jar contains 25 balls each bearing a different number from 1 through 25. You make 5 guesses as to what numbers will be pulled from the hat. You cannot guess the same number twice. Someone then pulls 5 distinct balls from the hat. What is the probability that **none** of your guesses coincide with a ball pulled from the hat? Report your answer either as a fraction in reduced terms or rounded to the nearest percentage.

Answer: $\frac{2584}{8855}$ OR 29%

(4) (10 points) What is the exact value of $\sum_{k=2}^{2019} \frac{1}{\log_k((2019!)^2)}$?

Answer: $\frac{1}{2}$

(5) (10 points) You are constructing a pyramid of coins. The first layer is a 100×100 square, the second layer is a 99×99 square and so on. The coins on the first layer of this pyramid are each worth 1¢, on the second layer 2¢, on the third 3¢ and so on. For which k is the k 'th layer of the pyramid is worth more money than any other layer?

Answer: 34

(6) (10 points) Let $f(x) = 2^x + 1$ and $g(x) = 2^{x+1}$. Find all real number solutions to $f(g(x)) = g(f(x))$. Simplify your answer.

Answer: $\log_2(\log_2(2 + \sqrt{3}))$