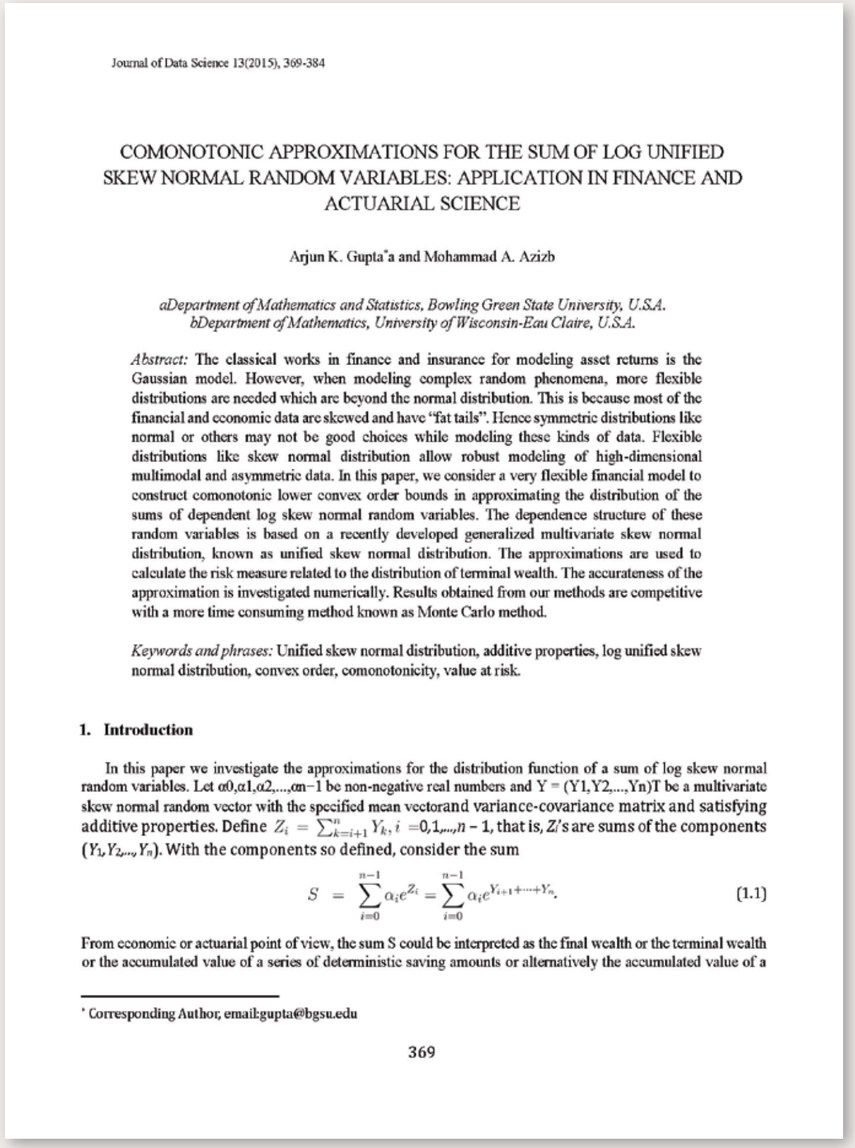

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COMONOTONIC APPROXIMATIONS FOR THE SUM OF LOG UNIFIED SKEW NORMAL RANDOM VARIABLES: APPLICATION IN FINANCE AND ACTUARIAL SCIENCE

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Abstract: The classical works in finance and insurance for modeling asset returns is the Gaussian model. However, when modeling complex random phenomena, more flexible distributions are needed which are beyond the normal distribution. This is because most of the financial and economic data are skewed and have "fat tails". Hence symmetric distributions like normal or others may not be good choices while modeling these kinds of data. Flexible distributions like skew normal distribution allow robust modeling of high-dimensional multimodal and asymmetric data. In this paper, we consider a very flexible financial model to construct comonotonic lower convex order bounds in approximating the distribution of the sums of dependent log skew normal random variables. The dependence structure of these random variables is based on a recently developed generalized multivariate skew normal distribution, known as unified skew normal distribution. The approximations are used to calculate the risk measure related to the distribution of terminal wealth. The accurateness of the approximation is investigated numerically. Results obtained from our methods are competitive with a more time consuming method known as Monte Carlo method.

Keywords and phrases: Unified skew normal distribution, additive properties, log unified skew normal distribution, convex order, comonotonicity, value at risk.

1. Introduction

In this paper we investigate the approximations for the distribution function of a sum of log skew normal random variables. Let $\alpha 0,\alpha 1,\alpha 2,\ldots,\alpha n-1$ be non-negative real numbers and $Y = (Y1,Y2,\ldots,Yn)T$ be a multivariate skew normal random vector with the specified mean vectorand variance-covariance matrix and satisfying additive properties. Define $Z_i = \sum_{k=i+1}^{n} Y_k, i = 0,1,\ldots,n-1$, that is, Z's are sums of the components $(Y_1,Y_2,\ldots,Y_n)$. With the components so defined, consider the sum

$$S = \sum_{i=0}^{n-1} \alpha_i e^{Z_i} = \sum_{i=0}^{n-1} \alpha_i e^{Y_{i+1}+\cdots+Y_n}. \quad (1.1)$$

From economic or actuarial point of view, the sum S could be interpreted as the final wealth or the terminal wealth or the accumulated value of a series of deterministic saving amounts or alternatively the accumulated value of a

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The normal distribution is widely used for a variety of applications in statistics and other fields because of its simplicity and accuracy in estimation. However, most financial and insurance data are skewed and have "fat tails." Hence, symmetric distributions may not be good choices for analyzing such data. In this paper, we use unified skew-normal density function to approximate the distribution of terminal wealth of a portfolio. We find that the results describing the terminal wealth obtained from approximations are competitive to those obtained from a more time consuming Monte Carlo method.