Sections 7.6:
L'Hôpital's Rule

L'Hospital's Rule is a method for computing the limits of functions of certain forms by using derivatives.

> **L'Hospital's Rule:** Suppose f and g are differentiable and $g'(x) \neq 0$ on an open interval that contains a except possibly at a. Furthermore suppose
>
> $$\lim_{x\to a} f(x) = 0 \text{ and } \lim_{x\to a} g(x) = 0$$
>
> or
>
> $$\lim_{x\to a} f(x) = \pm\infty \text{ and } \lim_{x\to a} g(x) = \pm\infty.$$
>
> Then
>
> $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$
>
> if the right hand limit exists or is $\pm\infty$.

Example 1:

$$\lim_{x\to 0} \frac{e^x - 1}{\sin x} = \lim_{x\to 0} \frac{e^x}{\cos x} = \frac{1}{1} = 1.$$

The above example is known as a limit of a function of the $\frac{0}{0}$ form.

Example 2: $\frac{\infty}{\infty}$ form.

$$\lim_{x\to\infty} \frac{e^x}{2x} = \lim_{x\to\infty} \frac{e^x}{2} = \infty.$$

Example 3: $0 \cdot \infty$ form.

$$\lim_{x\to 0^+} x \cdot \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{1}{-x^2}} = \lim_{x\to 0^+} (-x) = 0.$$

Example 4: $\infty - \infty$ form.

$$\lim_{x\to(\pi/2)^-} \sec x - \tan x = \lim_{x\to(\pi/2)^-} \frac{1-\sin x}{\cos x} = \lim_{x\to(\pi/2)^-} \frac{-\cos x}{-\sin x} = \lim_{x\to(\pi/2)^-} \frac{\cos x}{\sin x} = 0.$$

The three remaining L'Hospital's Rule Limit forms are the 0^0, ∞^0, and 1^∞ form. To calculate the limits by L'Hospital's rule we first take the logarithm and then compute the limit. We then exponentiate the limit of the logarithm to find the original limit.

$$A^B = e^{B \ln A} \text{ thus, } \lim A^B = e^{\lim B \ln A}.$$

Example 5: Determine $\lim_{x \to 0^+} x^x$.

We have $\ln x^x = x \ln x$ and $\lim_{x \to 0^+} x \ln x = 0$ by example 3. Thus $\lim_{x \to 0^+} x^x = e^0 = 1$.

Recommended Problems

pp 584-6, # 1, 2, 4, 5, 8, 9, 13, 16, 19, 21, 22, 24, 29, 34, 43, 45.