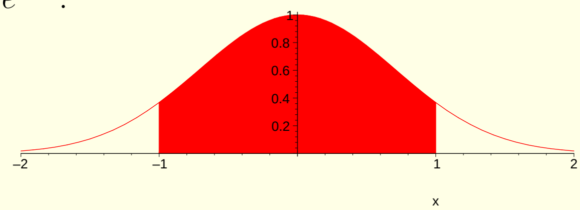
Sections 7.5: Integral Tables, Computer Algebra Systems, and Monte Carlo Integration

In the era of "classical" mathematics (before the pocket calculator and the personal computer) tables of indefinite integrals were compiled to facilitate computing integrals. In our textbook after the index a table of 141 integrals are listed. Now that personal computers are ubiquitous and Computer Algebra Systems, such as Maple and Mathematica, are available these tables have become less important. Here at UW-EC we utilize the CAS Maple quite extensively. Since computers are commonplace in the home and the workplace any person who uses mathematics should have familiarity with CAS systems. Hence the rational for introducing a laboratory experience into the calculus courses.

At first glance the fundamental theorem of calculus seems to be the solution to all integration, but it is not so. To apply the fundamental theorem an anti-derivative must exist. This does not always happen! Consider the function $f(x) = e^{-x^2}$.



As the above graph indicates $\int_{-1}^{1} e^{-x^2}\,dx$ exists, however the fundamental theorem does not help us compute this value as $f(x) = e^{-x^2}$ has no anti-derivative.

A method to approximate the area under a curve is one that utilizes the Average Value of a function. The average value of a function over an interval is the area under the function divided by the length of the interval (this is the same as computing the average speed of a car by dividing the distance travelled by the length of time of the trip). We thus have the following formula:

$$Avg = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Which yields:

$$\int_a^b f(x)\,dx = (b-a)Avg.$$

Now if we can estimate the average value of a function by statistical means then we can estimate the integral. We simply *randomly* select n values $\{x_i | i = 1..n\}$ and compute the average of the functional values.

$$Avg \approx \frac{1}{n}\sum_{i=1}^{n} f(x_i).$$

We thus have

$$\int_a^b f(x)\,dx \approx \frac{b-a}{n}\sum_{i=1}^{n} f(x_i).$$

This method is known as the *Monte Carlo* method as it utilizes the principles of chance (the random selection of points) to compute the approximation.

Recommended Problems

pp 576-7, # 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54.