I INTRODUCTION

Modern mathematics is couched in the language and steeped in the theory of sets. Sets are the heart and soul of mathematics. Sets should be well understood by any serious student of the subject. What we attempt to do here is to present the Zermelo Fraenkel Axioms for Set Theory and develop a model for those axioms. We call this model the *number model*, as our usual concept of numbers is a consequence of this model.

There is an initial temptation when developing a theory to define all of the terms. This temptation is quickly extinguished when we realize its futility. To define a new term we must define it in terms of previous terms; thus we are faced with two possibilities, either having circular definitions or having an infinite regression of definitions. Either case is unsatisfactory. Therefore we begin with undefined terms, or as I prefer "dictionary" defined terms. That is, definitions taken directly from a standard dictionary.

A **set** is a group of persons or things classed or belonging together. We may paraphrase this as: A set is a collection of objects. However, as we shall see later, not every collection can be regarded as a set. Collections, either sets or non-sets, are often referred to as **spaces** to avoid repetitive rhetoric.

The objects, persons, or things that make up the set we shall call **elements**. A single one of these objects is of course an element.

The aggregate of elements is the set. If a particular element is a member of the aggregate we say it is an element of the set. Let a represent a set and x one of its element. We express this fact with the notation, $x \in a$, and we read this as x is an element of a.

In developing an axiomatic discussion of sets it may be tempting to posit the existence of a set. This is troublesome for the following reason. In any well defined axiom system the axioms are chosen to be independent. An axiom is independent of the other axioms if a model can be constructed, using all other axioms, and replacing the axiom in question with another that is its denial or implies its denial. An axiom system is independent if all of its axioms are independent. If our axiom system includes an axiom that asserts the existence of a set, then to demonstrate its independence, we must construct a model that assumes the validity of all other axioms, which are statements about sets, and an axiom that denies the existence of a set. It is not necessary to posit the existence of a set since examples of concrete sets are ubiquitous. An examination of my right front pocket reveals a collection of keys, indeed a set exists.