THE $(n)$-SOLVABLE FILTRATION OF THE LINK CONCORDANCE GROUP AND MILNOR'S INVARIANTS

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ABSTRACT. We establish several new results about both the $(n)$-solvable filtration, \( \mathcal{F}^n_n \), of the set of link concordance classes and the $(n)$-solvable filtration of the string link concordance group. We first establish a relationship between Milnor’s invariants and links, \( \tau \), with certain restrictions on the link manifold bounded by \( M_2 \). Using this relationship, we can relate $(n)$-solvability of a link or string link with its Milnor’s $\tau$-invariants. Specifically, we show that if a link is $(n)$-solvable, then its Milnor’s invariants vanish for lengths up to \( 2^{n-1} \). Previously, there were no known results about the “other half” of the filtration, namely \( \mathcal{F}^n_m \). We establish the effect of the Hopf doubling operator on $(n)$-solvability and using this, we show that \( \mathcal{F}^n_n \) is nontrivial for links (and string links) with sufficiently many components. Moreover, we show that these quotients contain an infinite cycle subgroup. We also show that links and string links mod $(n)$-solvability is a torsors under the $(n)$-solvable filtration. We show that if a link is $(n)$-solvable, then its Milnor’s invariants will also vanish for lengths up to \( 2^{n-1} \). Lastly, we prove that the Goussarov filtration, \( \mathcal{G}^n_n \), of the set of link concordance classes is not the same as the $(n)$-solvable filtration.

1. Introduction

Much work has been done in the quest of understanding the $(n)$-solvable filtration. In particular, many have studied successive quotients of this filtration and some of their contributions can be found in [Chat99], [CHB98], [CHLO09], and [Har08].

For example, Harvey first showed that \( \mathcal{F}^n_n/\mathcal{F}^{n+1}_n \) is a nontrivial group that contains an infinitely generated subgroup [Har08]. She also showed that this subgroup is generated by boundary links (links with components that bound disjoint Seifert surfaces). Cockman and Harvey generalized this result

A link is a collection of knotted circles in 3-space. In this paper, I study the four dimensional relationship, called concordance on links, in order to find a group structure on the collection of links. Using higher order linking numbers, known as Milnor’s invariants, I investigate the relationship between links and the property of $(n)$-solvability, an algebraic approximation of a link being trivial in the concordance group.