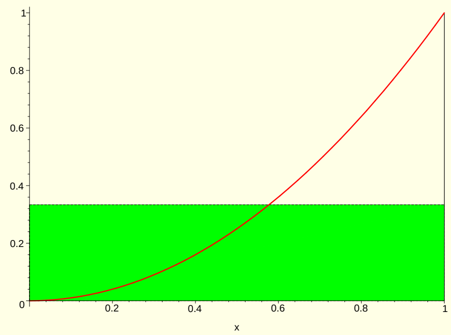
Sections 4.5:
The Mean Value and Fundamental Theorems

In this section we develop the Fundamental Theorem of Calculus. The fundamental theorem gives the relationship between the derivative and the definite integral.

Theorem: The Mean Value Theorem for Definite Integrals
If a function f is continuous on $[a, b]$, then at some point c in $[a, b]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$



The shaded area above the graph is equal to the unshaded area below the graph.

Theorem: The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t)\,dt$$

has a derivative at every point x in $[a, b]$, and

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Theorem: The Fundamental Theorem of Calculus, Part 2

If f is continuous on $[a, b]$, and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Example 1 Find dy/dx if $y = \int_0^x e^{t^2}\,dt$.

Solution $dy/dx = e^{x^2}$.

Example 2 Evaluate $\int_1^2 \frac{1}{x^2}\,dx$

Solution Since $\frac{d}{dx}(-\frac{1}{x}) = \frac{1}{x^2}$ we have

$$\int_1^2 \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_1^2 = -\frac{1}{2} - (-\frac{1}{1}) = -\frac{1}{2} + 1 = \frac{1}{2}$$

Recommended Problems

pp 383-6, #2, 4, 8, 12, 14, 40, 42, 52.