Sections 2.6: Implicit Differentiation

Recall that a function is a set of ordered pairs. We can indicate a function by the following notation: $\{(x, y)|y = f(x)\}$. $f(x)$ is often an algebraic expression that is a set of rules that tells us how to transform values of x into values of y. We often just write $y = f(x)$ or $f(x) =$ (some algebraic expression) to indicate a function.

Another way of indicating a function is $\{(x, y)|R(x, y)\}$ Where $R(x, y)$ is a mathematical expression that relates x and y, e.g. $\{(x, y)|x^2+y^2 = 1,\ y \geq 0\}$. In this situation we say that y is **implicitly** defined as a function of x, or simply $x^2 + y^2 = 1$ is an **implicitly defined** function.

We may differentiate an implicitly defined function by differentiating both sides of the equation and applying the chain rule in the terms that involve y. Using $x^2 + y^2 = 1$ as an example, we implicitly differentiate to get:

$$2x + 2y \cdot \frac{dy}{dx} = 0.$$

This simplifies to

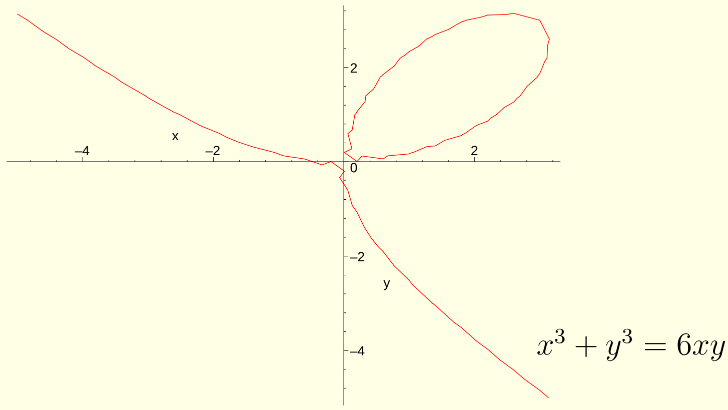
$$x + y\frac{dy}{dx} = 0.$$

or alternately written

$$x + yy' = 0.$$

We may solve for y' in terms of x and y to get $y' = \dfrac{-x}{y}$.

Example: Consider the relation $\{(x, y)|x^3 + y^3 = 6xy\}$.



The point $(3, 3)$ is on the graph. Find the equation of the tangent line to the graph at the point $(3, 3)$.

Solution: We implicitly differentiate to get

$$3x^2 + 3y^2y' = 6(y + xy').$$

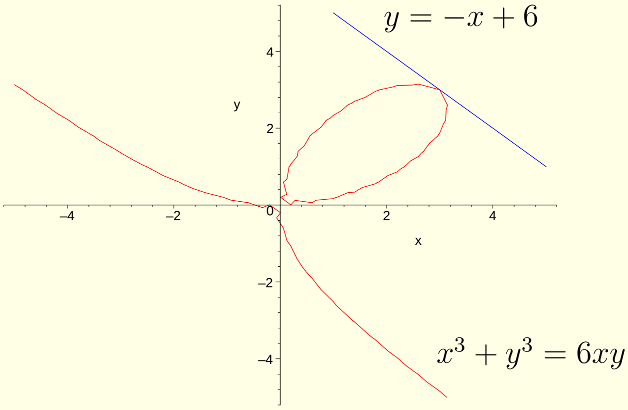
Now substitute $x = 3$ and $y = 3$ to get

$$3 \cdot 3^2 + 3 \cdot 3^2 \cdot y' = 6(3 + 3 \cdot y')$$

and solve for y'.

$$y' = -1$$

Thus the equation of the tangent line at $(3, 3)$ is $y = -x + 6$.



Recommended Problems

pp 204-6, #2, 12, 14, 18, 22, 28, 34, 38, 40, 44, 48, 52, 56.